

Medium Modifications of Mesons with charm ¹

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- Chiral partner sum rules
- Meson masses from Dyson-Schwinger – Bethe-Salpeter eqs.
(under construction)

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supported by BMBF & GSI-FE

Hadron physics and QCD sum rules

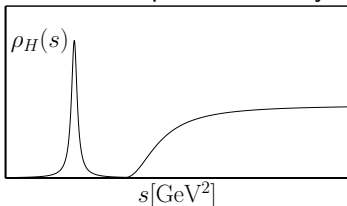
Current-Current Correlator

$$\Pi_{\mu\nu}^X(q) = i \int d^4x e^{-iqx} \langle T [j_{\mu}^{X,\tau}(x) (j_{\nu}^{X,\tau}(0))^{\dagger}] \rangle$$

Dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{\Delta\Pi(s)}{s-q^2}$$

hadronic properties encoded in spectral density



large euclidean momenta

Operator Product Expansion

$$= C_1(q) + C_2(q)\langle\bar{q}q\rangle + C_3(q)\langle\bar{q}q\sigma\mathcal{G}q\rangle + \dots$$

$$= \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$

The diagrams represent the OPE terms: a simple circle, a circle with two 'x' marks, and a circle with two 'x' marks and a vertical line through the center.

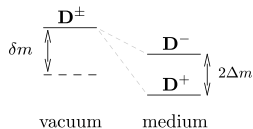
- medium dependence encoded in QCD condensates
- order parameter of chiral symmetry

Probing chiral symmetry restoration via the chiral condensate - light quark currents

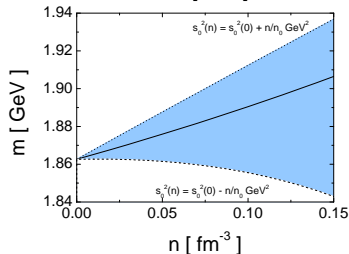
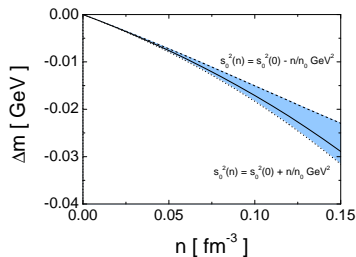
- $\langle \bar{q}q \rangle$ suppression in light-quark meson operator product expansion (e.g. ρ meson sum rules): $m_q \langle \bar{q}q \rangle$
- $\langle \bar{q}q \rangle$ influence only via assumptions/models: e.g.
 $\langle \bar{q}\Gamma q \bar{q}\Gamma q \rangle \propto \langle \bar{q}q \rangle^2$
→ fragile transfer to medium
- determination of other order parameters (e.g. four-quark condensates $\langle \bar{q}\Gamma q \bar{q}\Gamma q \rangle$) is model dependent

Probing chiral symmetry restoration via the chiral condensate - heavy-light quark currents ²

- $\langle \bar{q}q \rangle$ amplification due to heavy quark mass, e.g. D meson sum rules: $m_c \langle \bar{q}q \rangle$



- mass splitting is sensitive to:
 - $\langle q^\dagger q \rangle (= \frac{3}{2}n \propto \text{net quark density})$
 - $\langle \bar{q}q \rangle$, $\langle q^\dagger g \sigma \mathcal{G} q \rangle$
- mass center $D - \bar{D}$ is sensitive to
 - $\langle \bar{q}q \rangle$
 - continuum threshold



²[Hilger et al., Phys. Rev. C 79 025202 (2009)]

Light-quark chiral partner sum rules

- $n_f = 2$ Lagrangian:

$$\mathcal{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

for $m_{u,d} = 0$ invariant under transformation

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{-i\gamma_5 \frac{\bar{\lambda}}{2}} \bar{\Theta} \psi$$

$$j_\mu^{V,\tau}(x) = \bar{\psi} \gamma_\mu \tau \psi \longrightarrow j_\mu^{A,\tau}(x) = \bar{\psi} \gamma_5 \gamma_\mu \tau \psi$$

- chirally symmetric ground state \rightarrow current-current correlators

$$\Pi_{\mu\nu}^X(q) = i \int d^4x e^{-iqx} \langle T [j_\mu^{X,\tau}(x) (j_\nu^{X,\tau}(0))^\dagger] \rangle$$

are "blind" to parity

Weinberg-Kapusta-Shuryak sum rules ³

- finite density/temperature sum rules for vector-axial-vector currents of massless quarks

$$\int_0^\infty \frac{ds}{s} \Delta\Pi^{V-A} = F_\pi^2$$
$$\int_0^\infty ds \Delta\Pi^{V-A} = 0$$
$$\int_0^\infty ds s \Delta\Pi^{V-A} = -2\pi \langle \alpha_s O_\mu^\mu \rangle$$

- chiral condensate suppressed by light quark mass



³[S. Weinberg, Phys. Rev. Lett. 18 (1967) 507]

[J. Kapusta, E. Shuryak, Phys. Rev. D49 (1994) 4694]

Chiral partner sum rules for heavy-light mesons

- $n_f = 3$ Lagrangian with a "non-light" quark:

$$\mathcal{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{h} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_h \end{bmatrix} \right) \begin{pmatrix} u \\ d \\ h \end{pmatrix}$$

invariant under transformation

$$\psi = \begin{pmatrix} u \\ d \\ h \end{pmatrix} \rightarrow e^{-i\gamma_5 \frac{\bar{\lambda}}{2} \bar{\Theta}} \psi = \begin{pmatrix} u' \\ d' \\ h \end{pmatrix}$$

$$j_\mu^{V,\tau}(x) = \bar{\psi} \gamma_\mu \tau \psi \longrightarrow j_\mu^{A,\tau}(x) = \bar{\psi} \gamma_5 \gamma_\mu \tau \psi$$

Scalar and pseudoscalar mesons ⁴

Moments of the spectral difference for spin-0 heavy-light currents

$$j^{P,S} = \bar{q}_l(i\gamma_5)q_h:$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega \Delta \Pi^{P-S}(\omega) = -2m_h \langle \bar{q}q \rangle ,$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega^3 \Delta \Pi^{P-S}(\omega) = -2m_h^3 \langle \bar{q}q \rangle + m_h \langle \bar{q}g\sigma\mathcal{G}q \rangle - m_h \langle \Delta \rangle ,$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega^5 \Delta \Pi^{P-S}(\omega) = -2m_h^5 \langle \bar{q}q \rangle + 3m_h^3 \langle \bar{q}g\sigma\mathcal{G}q \rangle - 3m_h^3 \langle \Delta \rangle + \dots$$

- spectral difference driven only by order parameters of chiral symmetry breaking
- heavy quark mass amplifies influence of chiral condensate
- hierarchy of order parameters: $\langle \bar{q}q \rangle$,
 $\langle \bar{q}g\sigma\mathcal{G}q \rangle - \langle \Delta \rangle \propto \langle \bar{q}D_0^2q \rangle$

⁴[Hilger et al., J. Phys. G: Nucl. Part. Phys. 37 (2010) 094054]

[Hilger et al., Nucl. Phys. Proc. Suppl. 207-208 (2010) 277]

[Hilger et al., Phys. Rev. C, in print]

Vector and axialvector mesons

- currents not conserved
- longitudinal (L) and transversal (T) projection

$$\Pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi_T(q) + \frac{q_\mu q_\nu}{q^2} \Pi_L(q)$$

- mixing of quantum numbers ($m_I \rightarrow 0$)

$$\Pi_L^{V-A}(q) = -\frac{m_h^2}{q^2} \Pi^{S-P} - 2m_h \langle \bar{q}q \rangle$$

$$\Pi_T^{V-A}(q) = -\frac{m_h^2}{3q^2} \Pi^{P-S}(q) - \frac{1}{3} g^{\mu\nu} \Pi_{\mu\nu}^{V-A}(q) - \frac{2}{3} \frac{m_h}{q^2} \langle \bar{q}q \rangle$$

Moments of the spectral difference for spin-1 heavy-light currents:

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega \Delta \Pi_{\text{T}}^{\text{V}-\text{A}}(\omega) = -2m_h \langle \bar{q}q \rangle ,$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega^3 \Delta \Pi_{\text{T}}^{\text{V}-\text{A}}(\omega) = -2m_h^3 \langle \bar{q}q \rangle - \frac{4}{3} m_h \langle \Delta \rangle ,$$

$$\begin{aligned} \frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega^5 \Delta \Pi_{\text{T}}^{\text{V}-\text{A}}(\omega) &= -2m_h^5 \langle \bar{q}q \rangle + m_h^3 \langle \bar{q}g\sigma\mathcal{G}q \rangle \\ &\quad - \frac{11}{3} m_h^3 \langle \Delta \rangle + \dots \end{aligned}$$

- similar structure as in the P-S case
- order parameters: $\langle \bar{q}q \rangle$, $\langle \Delta \rangle$, $\langle \bar{q}g\sigma\mathcal{G}q \rangle$

[Hilger et al., Phys. Rev. C, in print]

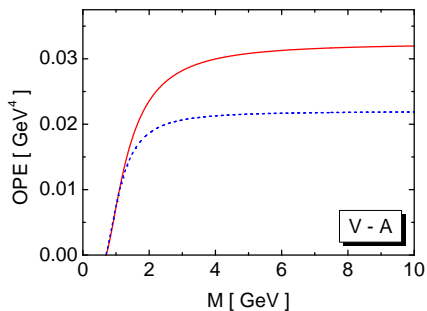
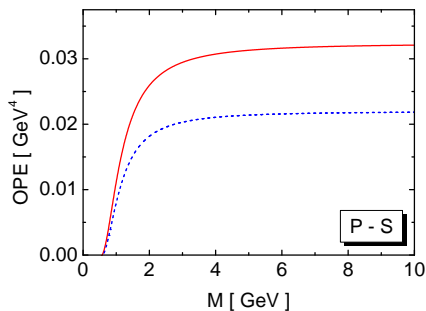
- Weinberg's first sum rule is recovered for $\tilde{\Pi}_T = \Pi_T/q^2$ (different analytic properties):

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega \Delta \tilde{\Pi}_T^{V-A}(\omega) = 0$$

- heavy quark limit:

$$\Pi_T^{V-A}(q) \Big|_{m_2^2 \gg |q^2|} \approx \Pi^{P-S}(q) \Big|_{m_2^2 \gg |q^2|} \approx -\frac{2}{m_2} \langle \bar{q}q \rangle$$

OPE for vacuum and medium



solid curve: vacuum
dashed curve: $n = n_0$

Dyson-Schwinger and Bethe-Salpeter

- Quark propagator (euclidean)

$$S_q^{-1}(p) = i\gamma \cdot p A(p) + B(p) = A(p) (i\gamma \cdot p + m(p)) \\ = (i\gamma \cdot p \sigma_v(p) + \sigma_s(p))^{-1}$$

- homogeneous Bethe-Salpeter boundstate and Dyson-Schwinger equation (euclidean) in rainbow-ladder truncation

$$\Gamma(P, p) = -\frac{4}{3} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu S(k_+) \Gamma(P, k) S(k_-) \gamma_\nu [g^2 D(p-l)]_{\mu\nu}$$

quark-gluon
vertex in rainbow
approximation

complex arguments

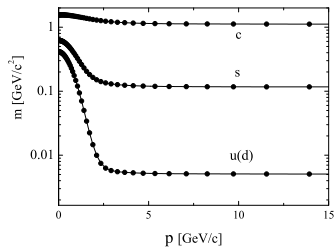
$$k_\pm = k + \eta_\pm P \\ \eta_+ + \eta_- = 1$$

gluon propagator
in ladder
approximation

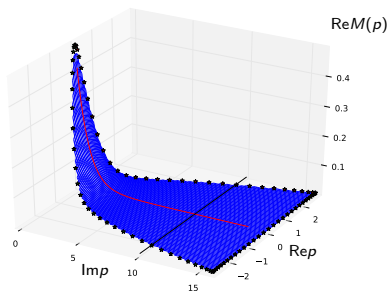
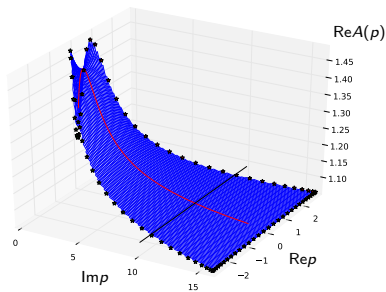
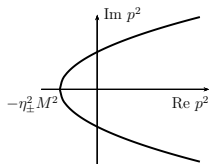
$$S_q^{-1}(p) = i\gamma \cdot p + \tilde{m} + \frac{4}{3} \int \frac{d^4 l}{(2\pi)^4} [g^2 D(p-l)]_{\mu\nu} \gamma_\mu S_q(l) \gamma_\nu$$

Dyson-Schwinger equation in the complex plane ⁵

- Dyson-Schwinger equation easily solved along real axis:

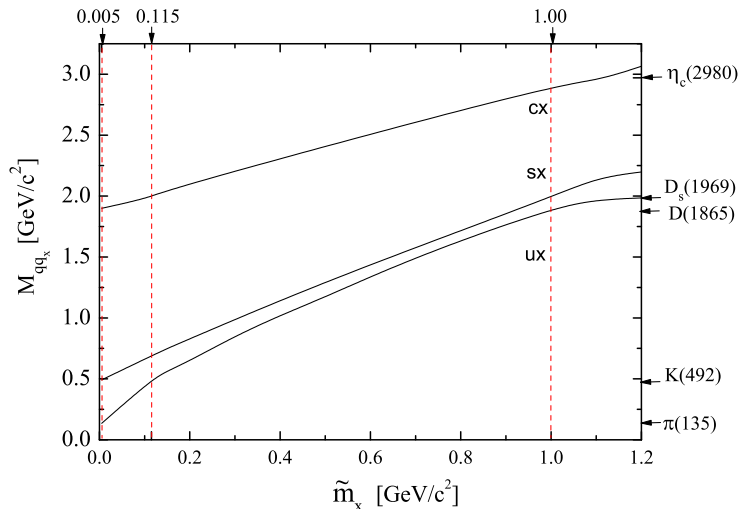


- Dyson-Schwinger equation within the complex plane:



⁵[Dorkin et al., Few Body Syst. 49:247-254, 2011]

Bethe-Salpeter equation results ⁶



⁶[Dorkin et al., Few Body Syst. 49: 247-254, 2011]

Summary

QCD sum rules

1. chiral partner sum rules:
chiral condensate \times heavy quark mass dominate the spectral difference of chiral partner
2.
 - coupled solution of Dyson-Schwinger and Bethe-Salpeter equation
 - investigation of analytic properties of the quark propagator is mandatory
 - extension of the method to finite densities/temperatures

Dyson-Schwinger–Bethe-Salpeter

first steps are done

goal: D spectroscopy in vacuum + medium