Chiral Symmetry and Medium Modifications of Mesons [1]

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- Chiral partner sum rules
- Meson masses from Dyson-Schwinger – Bethe-Salpeter eqs. (under construction)

[1] with B. Kämpfer, S. Leupold, S. Dorkin and L. Kaptari

supported by BMBF & GSI-FE

Hadron physics and QCD sum rules

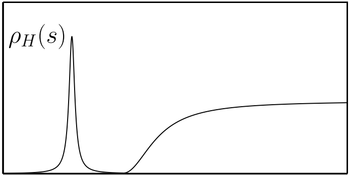
Current-Current Correlator

$$\Pi^{X}_{\mu\nu}(q) = i\int d^4x e^{-iqx}\langle \mathrm{T}\left[j^{X,\tau}_{\mu}(x)\left(j^{X,\tau}_{\nu}(0)\right)^{\dagger}\right]\rangle$$

Dispersion relation

$$\Pi(q) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}s \frac{\Delta\Pi(s,|\vec{p}|)}{s-q_0}$$

hadronic properties encoded in spectral density



$s[\mathrm{GeV}^2]$

large euclidean momenta

Operator Product Expansion

$$= C_1(q) + C_2(q)\langle\langle\bar{q}q\rangle\rangle + C_3(q)\langle\langle\bar{q}g\sigma\mathscr{G}q\rangle\rangle + \dots$$

=  + ...

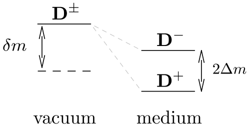
- medium dependence encoded in QCD condensates
- order parameter of chiral symmetry

Probing chiral symmetry restoration via the chiral condensate - light quark currents

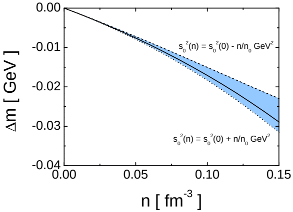
- $\langle \bar{q}q \rangle$ suppression in light quark meson operator product expansion (e.g. ρ meson sum rules): $m_q \langle \bar{q}q \rangle$
- $\langle \bar{q}q \rangle$ influence only via assumptions/models: e.g. $\langle \bar{q}\Gamma q \bar{q}\Gamma q \rangle \approx \langle \bar{q}q \rangle^2$
  $\rightarrow$ fragile transition to medium
- determination of other order parameters (e.g. four-quark condensates $\langle \bar{q}\Gamma q \bar{q}\Gamma q \rangle$) model dependent

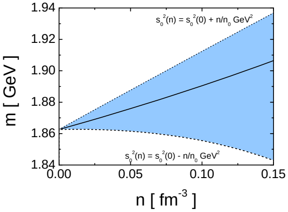
Probing chiral symmetry restoration via the chiral condensate - heavy-light quark currents [2]

- $\langle \bar{q}q \rangle$ amplification due to heavy quark mass, e.g. D meson sum rules: $m_c \langle \bar{q}q \rangle$



- mass splitting is sensitive to: $\langle q^\dagger q \rangle$ ($= \frac{3}{2} n \propto$ net quark density) $\langle \bar{q}q \rangle$, $\langle q^\dagger g \sigma \mathcal{G} q \rangle$
- mass center is very sensitive to $\langle \bar{q}q \rangle$ but also to the continuum threshold





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[2] [Hilger et al., Phys. Rev. C 79 025202 (2009)]

Light quark chiral partner sum rules

- SU($n_f = 2$) Lagrangian:

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

(approximately) symmetric under transformation

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{-i\gamma_5 \frac{\vec{\lambda}}{2}\vec{\Theta}} \psi$$

$$j_\mu^{\mathrm{V},\tau}(x) = \bar{\psi}\gamma_\mu \tau \psi \longrightarrow j_\mu^{\mathrm{A},\tau}(x) = \bar{\psi}\gamma_5\gamma_\mu \tau \psi$$

- chirally symmetric ground state → current-current correlators

$$\Pi_{\mu\nu}^{\mathrm{X}}(q) = i \int d^4x e^{-iqx} \langle \mathrm{T} \left[j_\mu^{\mathrm{X},\tau}(x) \left(j_\nu^{\mathrm{X},\tau}(0) \right)^\dagger \right] \rangle$$

are "blind" to parity

Weinberg-Kapusta-Shuryak sum rules

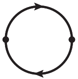
- finite density/temperature sum rules for vector–axial-vector currents of massless quarks

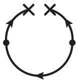
$$\int_0^\infty \frac{ds}{s} \Delta\Pi^{\mathrm{V-A}} = F_\pi^2$$

$$\int_0^\infty ds \Delta\Pi^{\mathrm{V-A}} = 0$$

$$\int_0^\infty ds\, s \Delta\Pi^{\mathrm{V-A}} = -2\pi \langle \alpha_s O_\mu^\mu \rangle$$

- chiral condensate suppressed by light quark mass





Chiral partner sum rules for heavy-light mesons

- SU($n_f = 3$) Lagrangian with a "non-light" quark:

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{h} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_h \end{bmatrix} \right) \begin{pmatrix} u \\ d \\ h \end{pmatrix}$$

still (approximately) symmetric under transformation

$$\psi = \begin{pmatrix} u \\ d \\ h \end{pmatrix} \to e^{-i\gamma_5 \frac{\vec{\lambda}}{2} \vec{\Theta}} \psi = \begin{pmatrix} u' \\ d' \\ h \end{pmatrix}$$

$$j_\mu^{\mathrm{V},\tau}(x) = \bar{\psi} \gamma_\mu \tau \psi \longrightarrow j_\mu^{\mathrm{A},\tau}(x) = \bar{\psi} \gamma_5 \gamma_\mu \tau \psi$$

Scalar and Pseudoscalar mesons [3] [4]

Moments of the spectral difference for spin-0 heavy-light currents:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega\Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h\langle\bar{q}q\rangle\ ,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^3\Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h^3\langle\bar{q}q\rangle + m_h\langle\bar{q}g\sigma\mathscr{G}q\rangle - m_h\,\langle\Delta\rangle\ ,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^5\Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h^5\langle\bar{q}q\rangle + 3m_h^3\langle\bar{q}g\sigma\mathscr{G}q\rangle - 3m_h^3\,\langle\Delta\rangle + \ldots\ .$$

- spectral difference driven only by order parameters of chiral symmetry breaking
- heavy quark mass amplifies influence of chiral condensate
- hierarchy of order parameters: $\langle\bar{q}q\rangle$, $\langle\bar{q}g\sigma\mathscr{G}q\rangle - \langle\Delta\rangle \propto \langle\bar{q}D_0^2 q\rangle$

[3] [Hilger et al., J. Phys. G: Nucl. Part. Phys. 37 (2010) 094054]
[4] [Hilger et al., Nucl. Phys. Proc. Suppl. 207-208 (2010) 277]

Vector and Axialvector mesons

- currents not conserved
- longitudinal (L) and transversal (T) projection

$$\Pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{\mathrm{T}}(q) + \frac{q_\mu q_\nu}{q^2}\Pi_{\mathrm{L}}(q)$$

- mixing of quantum numbers ($m_q \to 0$)

$$\Pi_{\mathrm{L}}^{\mathrm{V-A}}(q) = -\frac{m_h^2}{q^2}\Pi^{\mathrm{S-P}} - 2m_h\langle\bar{q}q\rangle$$

$$\Pi_{\mathrm{T}}^{\mathrm{V-A}}(q) = -\frac{m_h^2}{3q^2}\Pi^{\mathrm{P-S}}(q) - \frac{1}{3}g^{\mu\nu}\Pi_{\mu\nu}^{\mathrm{V-A}}(q) - \frac{2}{3}\frac{m_h}{q^2}\langle\bar{q}q\rangle$$

Moments of the spectral difference for spin-1 heavy-light currents:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega\,\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h\langle\bar{q}q\rangle\,,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^3\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h^3\langle\bar{q}q\rangle - \frac{4}{3}m_h\langle\Delta\rangle\,,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^5\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h^5\langle\bar{q}q\rangle + m_h^3\langle\bar{q}g\sigma\mathscr{G}q\rangle - \frac{11}{3}m_h^3\langle\Delta\rangle + \ldots\,.$$

- similar structure as in the P-S case
- order parameters: $\langle\bar{q}q\rangle$, $\langle\Delta\rangle$, $\langle\bar{q}g\sigma\mathscr{G}q\rangle$

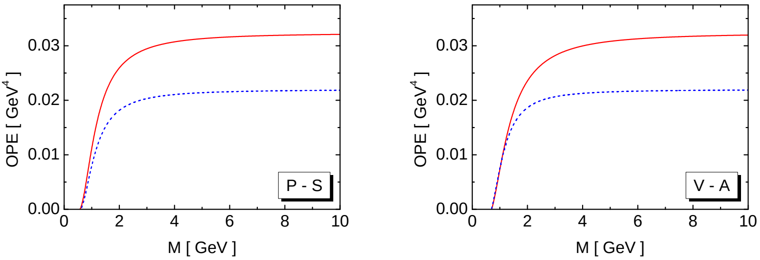
- Weinbergs first sum rule is recovered for $\tilde{\Pi}_{\rm T} = \Pi_{\rm T}/q^2$ (different analytic properties):

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\omega\Delta\tilde{\Pi}_{\rm T}^{\rm V-A}(\omega) = 0$$

- heavy quark limit:

$$\Pi_{\rm T}^{\rm V-A}(q)\Big|_{m_2^2\gg|q^2|} \approx \Pi^{\rm P-S}(q)\Big|_{m_2^2\gg|q^2|} \approx -\frac{2}{m_2}\langle\bar{q}q\rangle$$

Numerical results for vacuum and medium



Bethe-Salpeter and Dyson-Schwinger

- Quark propagator (euclidean)

$$S_q^{-1}(p) = i\gamma \cdot p\, A(p) + B(p) = A(p)\,(i\gamma \cdot p + m(p))$$
$$= (i\gamma \cdot p\, \sigma_v(p) + \sigma_s(p))^{-1}$$

- Bethe-Salpeter boundstate and Dyson-Schwinger equation (euclidean) in rainbow-ladder truncation

$$\Gamma(P,p) = -\frac{4}{3}\int \frac{d^4k}{(2\pi)^4}\, \gamma_\mu\, S(k_+)\Gamma(P,k)S(k_-)\, \gamma_\nu\, \left[g^2 D(p-l)\right]_{\mu\nu}$$

quark-gluon vertex in rainbow approximation

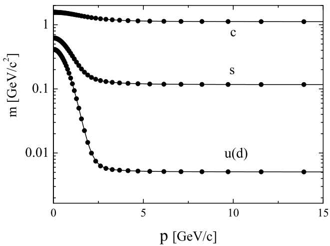
complex arguments $k_\pm = k + \eta_\pm P$, $\eta_+ + \eta_- = 1$

gluon propagator in ladder approximation

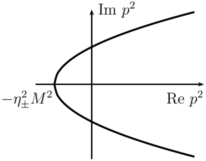
$$S_q^{-1}(p) = i\gamma\cdot p + \tilde{m} + \frac{4}{3}\int \frac{d^4l}{(2\pi)^4} \left[g^2 D(p-l)\right]_{\mu\nu} \gamma_\mu\, S_q(l)\, \gamma_\nu$$

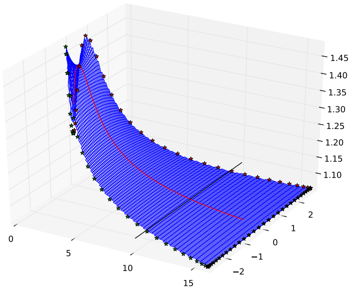
Dyson-Schwinger equation in the complex plane [5]

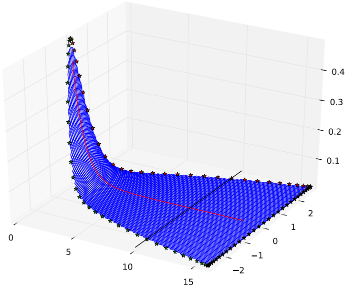
- Dyson-Schwinger equation easily solved along real axis:



- Dyson-Schwinger equation within the complex plane:

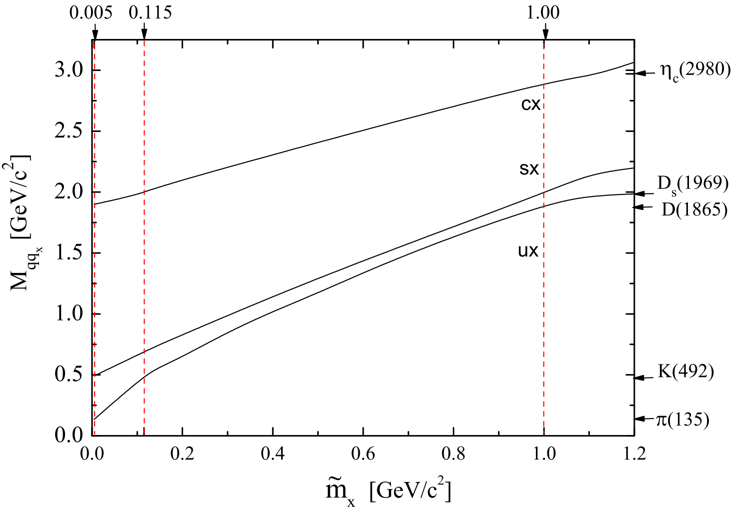






[5] [Dorkin et al., Few Body Syst. 49:247-254, 2011]

Bethe-Salpeter equation results [6]



[6] [Dorkin et al., Few Body Syst. 49: 247-254, 2011]

Summary

1. chiral partner sum rules: chiral condensate $\times$ heavy quark mass dominate the spectral difference of chiral partner
2. - coupled solution of Dyson-Schwinger and Bethe-Salpeter equation
   - investigation of analytic properties of the quark propagator is mandatory
   - extension of the method to finite densities/temperatures